Division **M** | *Mathematical Olympiads* **December 13, 2017** *for Elementary & Middle Schools* | Contest **2**

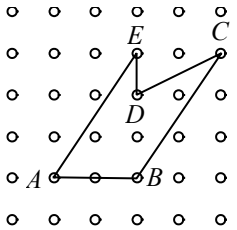
Directions to Students: After all questions have been read by your PICO, you will have 30 minutes to complete this contest. You may not have a pen or pencil in your hand while the PICO reads the set of questions to the class. Calculators are not permitted. All work is to be done on the pages provided. No additional scrap paper is to be used. Answers must be placed in the corresponding boxes in the answer column.

Name: _____

2A Reduce the complex fraction to a common fraction in lowest terms: $\dfrac{2}{3+\dfrac{4}{1-\dfrac{1}{5}}}$. [Hint: Start with $1-\frac{1}{5}$]

2B A palindrome reads the same forwards and backwards. The number 2017102 is a 7-digit palindrome. Let *A* represent the least palindrome greater than 2017102. Let *B* represent the greatest palindrome less than 2017102. Find *A* – *B*.

2C Thirty-six points are arranged in a unit-square array as shown. Figure *ABCDE* is composed entirely of straight-line segments with vertices *A*, *B*, *C*, *D*, and *E*.



Find the number of degrees in the sum of the interior angles of figure *ABCDE*. [*The interior angle at D is a reflex angle whose measure is greater than 180°.*]

Name: _____

Answer Column
2A
2B
2C
2D
2E
Do Not Write in this Space. *For PICO's Use Only.* *SCORE:*

– Page may be folded along dotted line. –

2D Alexi opens his favorite mathematics puzzle book and notes that the product of the page numbers facing him is 1,332. Find the sum of these two page numbers. [Note: All pages are numbered consecutively.]

– Page may be folded along dotted line. –

2E What is the greatest whole number less than 1000 that:
(1) can be expressed as the sum of two consecutive whole numbers, and
(2) can be expressed as the sum of three consecutive whole numbers, and
(3) can be expressed as the sum of five consecutive whole numbers?

Division M — Mathematical Olympiads for Elementary & Middle Schools — December 13, 2017 — Contest 2

SOLUTIONS AND ANSWERS

2A 1/4

2B 2000

2C 540°

2D 73

2E 975

2A **METHOD 1** *Strategy: Combine numbers starting at the bottom.*

Start with $1-\frac{1}{5}=\frac{4}{5}$. Then $\frac{4}{4/5}=4\times\frac{5}{4}=5$. Next, 3 + 5 = 8 and $\frac{2}{8}=\frac{1}{4}$.

METHOD 2 *Strategy: Use multiplication to simplify the complex fraction.*

Multiply $\frac{4}{1-\frac{1}{5}}$ by the number 1 in the form of $\frac{5}{5}$ to get $\frac{20}{5-1}$ = 5.

Then $\frac{2}{3+5}=\frac{2}{8}=\frac{1}{4}$.

FOLLOW UP: Solve for N: $\frac{1}{1-\frac{1}{N}}=3$. [3/2]

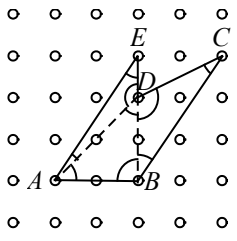
2B *Strategy: Reason using place value.*

Changing the digit in the units column forces the digit in the millions column to change. Likewise, changing the digits in the tens or hundreds columns changes the digits in the hundred-thousands or ten-thousands columns. Each of these changes would bring the result further from the original number than changing the number in the thousands column. Thus the two closest palindromes to 2017102 would be 2018102 and 2016102. These numbers are **2000** apart.

FOLLOW UPS: (1) How many 4-digit palindromes are there between 2017 and 3017? [10] *(2) Consider using letters instead of numbers to form a 3 letter "word" that is a palindrome (e.g. BQB). If a word consists of only the vowels A,E,I,O,U, how many 3-letter palindromes can be formed?* [25]

2C **METHOD 1** *Strategy: Dissect the figure into triangles.*

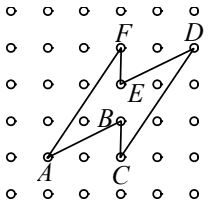
Draw $\overline{AD}$ and $\overline{BD}$ to form three triangles. The sum of the angles in the three triangles equals the sum of the five angles in *ABCDE*. The sum of the angles in any triangle is 180°, so the sum of the interior angles in *ABCDE* is 3(180°) = **540°**.



METHOD 2 *Strategy: Dissect the figure into 2 triangles.*

Draw $\overline{BD}$ creating ΔAEB and ΔDBC. The sum of the angles in the two triangles is 2(180°) = 360°. Then add the measure of straight $\angle EDB$ (180°), which is part of reflex $\angle EDC$, to get the total for the sum of the interior angles for *ABCDE* to equal 360 + 180 = 540°.

FOLLOW UP: Find the number of degrees in the sum of the interior angles of figure ABCDEF. [720°]



2D **METHOD 1** *Strategy: Use the idea that the square root will be close to the answer .*
The two page numbers will be consecutive with one slightly less than the square root and the other slightly larger. Using any convenient method, determine that the square root of 1332 is slightly larger than 36. The two page numbers are 36 and 37. The sum of these numbers is 36 + 37 = **73**.

METHOD 2 *Strategy: Find two consecutive numbers whose product has a units digit of 2.*
Determine by approximation that the pages are in the 30's. Consecutive numbers whose product ends in 2 are 3 × 4 and 6 × 7. The possible products are 33 × 34 = 1122 and 36 × 37 = 1332. The sum 36 + 37 = 73.

FOLLOW UPS: (1) If the sum of the last 4 pages in the puzzle book is 786, find the last page number. [198]
(2) The sum of two non-consecutive page numbers is 45, and their difference is 27. Find their product. [324]

2E *Strategy: Consider the properties of sums.*

The sum of two consecutive numbers must be odd. The sums of three consecutive numbers and of five consecutive numbers must be multiples of 3 and of 5 respectively. (Consider the 3 consecutive numbers n, $n + 1$, and $n + 2$. Their sum is $3n + 3$, which is a multiple of 3. A similar conclusion can be drawn for five numbers.) The number we want is the greatest odd multiple of both 3 and 5 (i.e. an odd multiple of 15) smaller than 1000. Odd multiples of 5 end in a 5. Look at 995 (not a multiple of 3), 985 (not a multiple of 3), and 975 (a multiple of 3). Since **975** is also a multiple of 3 and 5 it is the greatest number less than 1000 that satisfies all three conditions.

Verify: **975** = 487 + 488 and **975** = 324 + 325 + 326 and **975** = 193 + 194 + 195 + 196 + 197.

NOTE: *Other FOLLOW UP problems related to some of the above can be found in our three contest problem books and in "Creative Problem Solving in School Mathematics." Visit www.moems.org for details and to order.*